

Using genetic programming to optimise pricing rules for a double auction market

Steve Phelps
Peter McBurney
Dept of Computer Science
University of Liverpool
Chadwick Building
Liverpool L69 7ZF, UK.
sphelps,peter@csc.liv.ac.uk

Elizabeth Sklar
Dept of Computer Science
Columbia University
1214 Amsterdam Avenue
New York, NY 10027, USA.
sklar@cs.columbia.edu

Simon Parsons
Dept of Computer and
Information Science
Brooklyn College
2900 Bedford Avenue
Brooklyn, NY 11210, USA.
parsons@sci.brooklyn.cuny.edu

ABSTRACT

The mechanism design problem in economics is about designing rules of interaction for market games so they yield a globally desirable result in the face of self-interested agents. This problem, which is of importance for ecommerce since much ecommerce is carried out through auctions, can be extremely complex. Traditionally, economists have tried using game theory and other formal methods to construct suitable mechanism rules. However, analytical methods typically oversimplify the problem and so the resulting rules are not necessarily robust. In this paper, we report on an alternative approach which we hope will eventually yield more robust solutions. Our methodology views mechanism design as a multi-objective optimisation problem and addresses the problem using genetic programming.

1. INTRODUCTION

The growth in ecommerce has been matched by the growth in the use of auctions as mechanisms for identifying trading partners and setting transaction prices. This is not just true of consumer transactions as carried out on, eBay¹ but also of commercial transactions, with increasing numbers of companies choosing to carry out procurement operations and to set up supply chains by using auctions. This in turn has increased interest in the problem of *mechanism design* for auctions—the business of determining the form that an auction should take and the rules by which it is conducted.

Economists, and latterly computer scientists, have had considerable success in applying techniques from game theory to the design of auction-based markets for deregulated commodity markets, for example California's deregulated electricity market [6, 19] and the sale of government assets like electromagnetic spectrum for mobile phones [21,

¹ www.ebay.com

20]. Alvin Roth [29] has suggested that this is akin to an engineering process in which economists design the rules of a market mechanism in order to meet particular socio-economic requirements (for example maximising the efficiency of allocating commodities in a market).

The engineering of auction mechanisms—a subfield of computational economics²—is of particular importance to agent-based electronic commerce and multi-agent systems in general. Ecommerce has enabled consumers to act as price-makers instead of just price-takers in large auction-based markets and has stimulated the use of personalised bidding agents to empower those consumers even more. In addition, auction mechanisms are seen as a promising means of solving many distributed resource-allocation problems in multi-agent systems and grid technology.

One approach to computational economics is to use techniques from machine learning to explore the space of possible ways in which agents might act in particular markets. For example, reinforcement learning has been used to explore bidding patterns in auctions [25, 29] and to establish the ways in which price-setting behaviour can affect consumer markets [32]. Another approach is to use techniques from evolutionary computing, such as genetic programming (GP) [18]. Our earlier work has explored the use of co-evolutionary GP to determine auction mechanism rules automatically [26, 27].

In that work, mechanism rules and bidding strategies were encoded and co-evolved in ways that sought to maximise overall market efficiency and the profits of individual agents. Here, we focus on the multi-objective optimisation issues inherent in the mechanism design problem.

The rest of the paper is organised as follows. In Section 2, we describe the standard view of n -player games and introduce the perspective we will take in this paper. Then in Section 3, we discuss in detail the problem that we have been experimenting upon. We present two sets of data: first, an attempt to map the fitness landscape using a standard class of auction pricing rules—the k -double auction pricing rule—(see section 4.1); and second, an experiment in which search the space of possible pricing rules by using genetic programming (section 4.2). We close with a discussion and summary.

² *Computational economics* is the study of economic systems viewed as evolving systems of autonomous agents [33].

2. EQUILIBRIA FOR N -PLAYER GAMES

When evaluating a mechanism design, the designer must take into account the set of trading strategies that are likely to be played by agents trading according to the mechanism under consideration. Deriving the set of the strategies likely to be played for a particular market game, that is “solving” the game, is a non-trivial problem in the general case. This is because there is often no clear *dominant* strategy which constitutes best play; rather the best strategy to play depends entirely on the strategies played by other agents. Nash [24] identified a class of solutions in which the strategy adopted by any given agent is a best-response to the best-response strategies adopted by all other agents, and proved that all n -player, non-zero-sum games admitted such solutions. The situation in which all agents adopt this “best-response to best-responses” strategy is the famous Nash equilibrium.

Nash equilibrium, and its generalisation—the Bayesian Nash Equilibrium [15] in which each agent constructs its best-response based on its belief about whatever information it does not have—are widely adopted in theoretical economics. Thus when evaluating an economic mechanism, the designer typically computes the (Bayesian) Nash equilibria of strategies for the given mechanism; and this forms the basis of predictions about how people will actually behave under the rules of this mechanism. The designer can then analyse market outcomes in equilibria and quantitatively assess, for example, the likely effect on overall market-efficiency that a given change in the mechanism rules will yield. Thus the role of the designer is to ensure that the Nash equilibria correspond to situations in which high market efficiency is obtained, and the process of doing this can be an analytic or a computational one.

However, there are a number of difficulties with establishing Nash equilibria. For a start, obtaining Nash equilibria analytically is hard. In the case of the k -double-auction, for example, analytical techniques have yet to yield a solution except for cases with unrealistic simplifying assumptions [9]. As a result, economists have turned to computational approaches, in particular learning techniques, to try and compute Nash equilibria, as in [12]. These approaches have their own problems. Jordan [16] shows that there are simple games in which the strategies learnt do not converge to Nash equilibria, and, as Dekel *et al.* [7] point out, the existence of steady states that are not Nash equilibria is a good reason to question why only Nash equilibria are considered important.

These are all practical difficulties, but there are also theoretical difficulties. Empirical evidence shows that human agents often fail to coordinate on Nash-equilibria for very simple games whose solution is easily derivable under bounded-rationality assumptions [14, 1]. This suggests that it is not appropriate to center the efficiency of a given mechanism around the assumption that players will adopt Nash equilibria if some of those players will be human.

These difficulties with the standard theory of games have led to the development of a field known as *cognitive game theory* [8], in which models that capture elements of human learning play a central role in explaining and predicting strategic behaviour. Erev and Roth [30] show how simulations of agents equipped with a simple reinforcement learning algorithm can explain and predict the experimental data observed when human agents play a diverse range of trading games. Such *multi-agent reinforcement learning* models

form the basis of the approach to automated mechanism design that we explore here. Rather than computing the theoretical equilibria for a given point in the mechanism search space, we run a number of multi-agent simulations using agents equipped with a learning algorithm that determines their bidding strategies and take the final strategies as defining the equilibrium.

The approach we have been following makes use of this cognitive approach and generalises the search for steady states from that generally used in the literature. It is common to view mechanism design as the search for a mechanism that optimises a single parameter—market efficiency for example. We, in contrast, consider mechanism design to be a *multi-objective optimization* problem in which we simultaneously maximise several parameters—market efficiency and trader market power being two we consider in this paper. The difficulty in doing this lies in simultaneously maximising as many dimensions as possible.

Note that we are not attempting to find theoretically optimal strategies for our agents. Rather, we are attempting to predict how bounded-rational agents, who have no prior knowledge of an equilibrium solution nor the means to calculate one, might actually play against the mechanism we are (automatically) designing. For this reason, we chose to use the Roth-Erev algorithm [30], since it forms the basis of a *cognitive model* of how people actually behave in strategic environments. In particular it models two important principles of learning psychology:

- *Thorndike’s law of effect*—choices that have led to good outcomes in the past are more likely to be repeated in the future; and
- *The power law of practice*—learning curves tend to be steep initially, and then flatten out.

The Roth-Erev algorithm belongs to a class of game-playing models known as “stimuli-response” models. These models have much in common with the replicator dynamics model of evolutionary game theory [2], and as in evolutionary game theory, the stable asymptotic behaviour of a multi-agent simulation using the Roth-Erev learning model can be interpreted similarly to the Nash-equilibrium of classical game theory or the evolutionary-stable-strategy of evolutionary game theory; stable states constitute strategy sets that are hard-to-leave and are likely to persist once they are reached, even when we consider agents who are not using the actual Roth-Erev learning algorithm to form their strategy”.

In the remainder of this paper, we describe how we have used these ideas to carry out some experiments in automated mechanism design in the setting of a particular kind of commodity market.

3. EXPERIMENTAL SETUP

We can think of the commodity market as an iterated game between three types of agents: *sellers*, *buyers* and *auctioneers*. Each iteration of the game involves three steps. First, the traders (the set of all buyers and sellers) make a move. These moves collectively indicate how many units they want to trade and at what price they wish to trade. The auctioneer then moves, matching traders based on the last moves that traders made. Finally the traders either accept or reject the matches suggested by the auctioneer.

This scenario stems from [25] (hereafter referred to as *NPT*). A more detailed description of our interpretation can be found in [26]. In this scenario, a number of traders buy and sell electricity in a discriminatory-price³ continuous double auction [10]. Every trader assigns a value for the electricity that they trade; for buyers this is the price that they can obtain in a secondary retail market and for sellers this reflects the costs associated with generating the electricity. Here this value is considered *private*; because traders are always trying to make a profit themselves, sellers are not willing to reveal how little they might accept for units of electricity and buyers are not willing to reveal how much they might pay for units of electricity. Trade in electricity is also affected by capacity constraints; every trader has a finite maximum capacity of electricity that they can generate or purchase for resale.

The market proceeds in rounds of trading. In the first step within a round, sellers have the choice between issuing an “ask” or a “pass”, and buyers have the choice between issuing a “bid” or a “pass”.⁴ In the second step, auctioneers can either match buyers and sellers (or more accurately the bids and asks issued by buyers and sellers), `match(buyer, seller, price, quantity)`, or they can “pass”. Note that during an auctioneer’s turn, she can make either one or more matches or a single pass. The market proceeds until a set number of auction rounds is reached.

The key to the operation of the market is the auctioneer’s job of matching buyers and sellers, based on their current bids and asks, and setting the *trade price* at which units of capacity are traded. In our work, the matching process is carried out using the 4-heap algorithm [35]. The rule for determining the trade price is what we are trying to evolve.

In our experiments, the number of sellers, *NS*, is the same as the number of buyers, *NB*, and there is one auctioneer (*A*). All traders have a capacity of 10 units. Traders are equipped with the modified version of the modified Roth-Erev (MRE) learning algorithm described in [25]. The MRE algorithm is calibrated with three parameters: a scaling parameter $s(1)$, a recency parameter r and an experimentation parameter e (for a discussion of these parameters see [25]). The values used here were $s(1) = 1$, $r = 0.1$ and $e = 0.2$.

Our design objective is to increase the efficiency of the market, whilst simultaneously keeping the market-power, the degree to which they can control the trade price, of both buyers and sellers to a minimum—we want to increase global profit but without giving unfair advantage to either buyers or sellers. To do this we need to measure efficiency and market power and we have adopted the three variables used in *NPT*, namely: *market efficiency*, *seller market-power* and *buyer market-power*. Here we present a brief summary of these variables (refer to [25] for details). Market efficiency, *EA*, is defined as:

$$EA = 100 \left(\frac{PBA + PSA}{PBE + PSE} \right) \quad (1)$$

PBA and *PSA* are the profits that the buyers and sellers, respectively, actually make. *PBE* and *PSE* are the profits theoretically available to buyers and sellers, respectively, in

³In *uniform* price auctions, all trades in any given auction round happen at the same price. In *discriminatory* price auctions of the kind we have here, different trades in the same auction round occur at different prices.

⁴“Bid” and “Ask” are standard double auction terminology.

an market where all traders bid truthfully and an optimal allocation is made. (We can, of course, compute the result of agents bidding truthfully since we have access to their private values outside the simulation.)

Buyer market-power, *MPB*, is defined as the difference between the actual profits of buyers, *PBA*, and the potential equilibrium profits *PBE* for buyers, expressed as a ratio of the equilibrium profits.

$$MPB = \frac{PBA - PBE}{PBE} \quad (2)$$

Seller market-power is computed in the same way:

$$MPS = \frac{PSA - PSE}{PSE} \quad (3)$$

Market efficiency, *EA*, tracks how good our mechanism is at generating *global profit*, whereas the market-power indices, *MPB* and *MPS* track to what extent each group is better or worse off compared to the ideal market.

Strategic buyer market power *SMPB* measures the difference between the actual profits of the buyers and the profits they would get if they bid truthfully in the current market (as opposed to the ideal market assumed when calculating equilibrium profits), expressed as a fraction of equilibrium profits:

$$SMPB = \frac{PBA - PBT}{PBE} \quad (4)$$

Strategic seller market-power is computed in the same way:

$$SMPS = \frac{PSA - PST}{PSE} \quad (5)$$

Zero strategic market-power values strongly suggest that the mechanism is strategy proof—that is there is no way for a given trader to systematically generate profits at the expense of the other traders.

We normalise each variable by mapping it onto the range [0, 1], where 1 represents the optimal value of a variable and 0 represents the worst value. Variables are mapped using the following functions:

$$\widehat{EA} = \frac{EA}{100} \quad (6)$$

$$\widehat{MPB} = \frac{1}{1 + MPB} \quad (7)$$

$$\widehat{MPS} = \frac{1}{1 + MPS} \quad (8)$$

$$\widehat{SMPB} = \frac{1}{1 + SMPB} \quad (9)$$

$$\widehat{SMPS} = \frac{1}{1 + SMPS} \quad (10)$$

Given these, our aim is to perform a multi-objective optimisation of efficiency and market power. For our initial experiments we combine our different objectives in a simple linear sum with fixed weightings and optimise the scalar fitness value for the particular case where we give equal weighting to efficiency and market-power. Since we have two measures of market power we have two values of optimise:

$$F = \frac{\widehat{EA}}{2} + \frac{\widehat{MPB} + \widehat{MPS}}{4}$$

$$V = \frac{\widehat{EA}}{2} + \frac{\widehat{SMPB} + \widehat{SMPS}}{4}$$

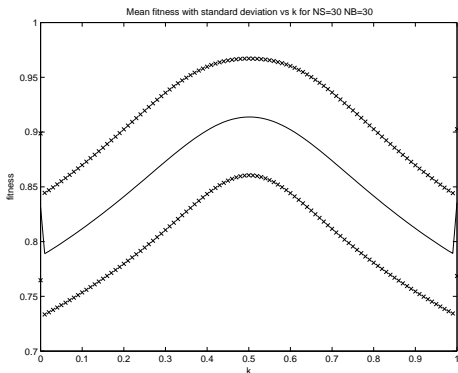


Figure 1: Fitness F (with standard deviation) plotted against k for a market with 60 traders.

In future work we will use multi-objective evolutionary algorithms to explore the full pareto frontier of these problems.

For now, we have restricted our search of the mechanism design space to the *transaction pricing rule*, which sets the price of any given transaction as a function of the *bid* and *ask* prices submitted by buyers and sellers respectively. *NPT* uses a discriminatory-price k -double-auction transaction pricing rule [31], in which a different transaction price is awarded for each matched bid-ask pair in the current auction round. The price is set according to the following function:

$$p_t = kp_a + (1 - k)p_b \quad (11)$$

where p_t is the transaction price, p_a is the ask price, p_b is the bid price and k is a parameter that can be adjusted by the auction designer. In the original *NPT* experiments k is taken to be 0.5.

Our aim is to investigate if there are alternatives to the k -double-auction rule that perform well, not necessarily under equilibrium conditions, but when agents play Roth-Erev derived strategies; that is, adaptive strategies derived from a *cognitive* model of human game playing.

In our experiments, we consider the space of all possible pricing rules that are functions of p_a and p_b . We represent each function as a Lisp s-expression, and we use Koza’s genetic programming [18] to search this space. Individual mechanisms are compared according to the criteria represented by F in order to judge their fitness, thus we are using genetic programming to solve a multi-objective optimisation problem. We return to the full details of our GP experiment in Section 4.2.

One might ask why we are using genetic programming to search such a vast space, when we could simply restrict attention to the k -double-auction pricing rule, and search for optimal values of k . The reason we use genetic programming is that we see this as a general method of representing *arbitrary* mechanism rules, not just those that can be neatly parameterised. In this particular case, we have chosen an aspect of the auction design that can be simply parameterised, so that we can compare the performance of the genetic programming search against a brute-force search of different values of k . In the following section we use a brute-force search of k to get an approximate view of the fitness landscape that our genetic programming search will encounter. In future work, we will use genetic programming to search

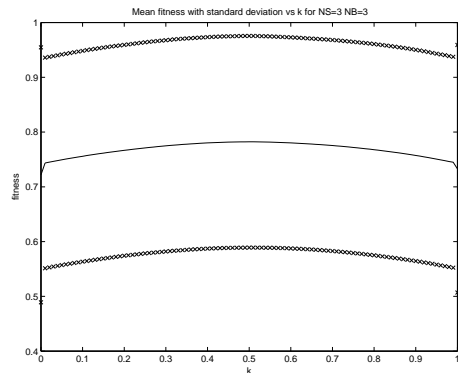


Figure 2: Fitness F (with standard deviation) plotted against k for a market with 6 traders.

for additional rules governing the auction mechanism, for example rules governing allowable bids and rules governing the matching mechanism.

4. EXPERIMENTAL RESULTS

In this paper we report on two aspects of the experimental work we have been carrying out within the electricity market scenario. First we describe work to map out the fitness landscape in which the pricing rule is evolving. We do this by assuming a k -double auction and then calculating the efficiency of the market for different values of k . Second, we describe an experiment in which the pricing rule was free to evolve and show that it converged on the k -double auction rule with $k = 0.5$.

4.1 Mapping the landscape

We carried out two mappings of the fitness landscape, and for both we did this by running auctions with 100 different values of k at increments of 0.01.

In the first mapping, each auction was run for 100 rounds, and for each value of k we ran 1000 auctions each with a different supply and demand schedule. These schedules were constructed by assigning each agent a random private value from a uniform distribution in the range [30, 1000]. The market variables under observation are averaged over these 1000 different schedules. Figure 1 shows the mean fitness measure F for each value of k when the market consists of 60 traders (30 buyers and 30 sellers) and Figure 2 shows the mean fitness measure F for each value of k when the market consists of 6 traders (3 buyers and 3 sellers)

In the second mapping we looked at fitness measure V . This time, concerned by the size of the standard deviations in the first mapping, we ran each auction for 1000 rounds and used 100,000 supply and demand schedules. The results of this mapping is given in Figures 3 and 4 for 60 traders (30 buyers and 30 sellers) and 12 traders (6 buyers and 6 sellers) respectively. For the second mapping we also looked at the measures of strategic buyer and seller market power. These are shown in Figures 5 and 6 and suggest that overall strategic market power (the sum of the buyer and seller figures) is approximately zero for $k = 0.5$.

These mappings at different values of k give us an idea of the fitness landscape for the electricity scenario when using our measures of fitness. A qualitative interpretation of

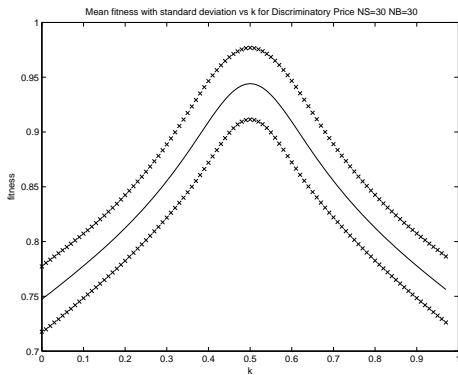


Figure 3: Fitness V (with standard deviation) plotted against k for a market with 60 traders.

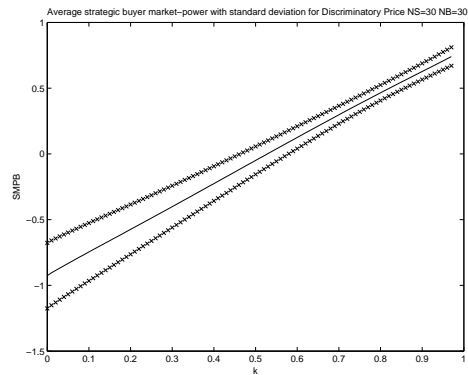


Figure 5: Strategic buyer market power plotted against k for a market with 60 traders.

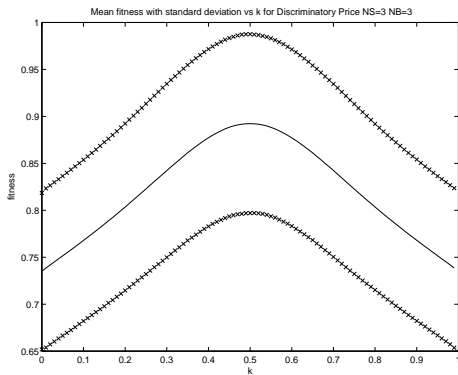


Figure 4: Fitness V (with standard deviation) plotted against k for a market with 12 traders.

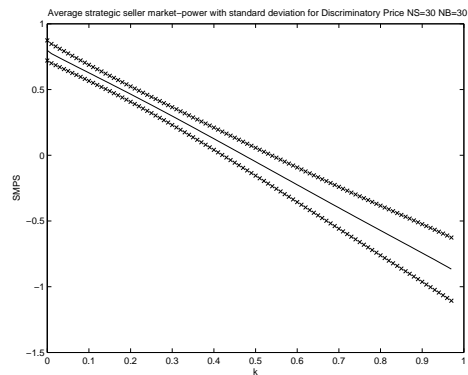


Figure 6: Strategic seller market power plotted against k for a market with 60 traders.

this data would suggest that values of k close to 0.5 should be selected by any technique that is applying the k -double auction rule and attempting to learn the best value of k while using our fitness measures. In the classical analysis of the k -CDA [35] it can be demonstrated that the auction is strategy proof for buyers for $k=0$, and is strategy proof for sellers for $k=1$, but is not simultaneously strategy proof for both groups. Our results, however, suggest that the k -CDA is "statistically" strategy proof for both buyers and sellers for $k=0.5$.

4.2 Evolving pricing rules

Having established the fitness landscape assuming the k -double auction rule, we then set out to search the entire space of possible pricing rules using genetic programming. We represented each rule as a Lisp s-expression, and we used Koza's basic genetic programming [18] with the parameters given in Table 1 to search this space. We made use of a Java-based evolutionary computation system called ECJ.⁵ ECJ implements a strongly-typed GP [23] version of Koza's [18] original system. For the GP experiments in this paper, the standard Koza parameters were used in combination with the standard Koza GP operators, with the addition of a small amount of parsimony pressure (applied with

probability 0.005) in order to counter the effects of GP code bloat.

Our function-set consisted of the terminals *ASKPRICE* and *BIDPRICE*, representing p_a and p_b respectively, together with the standard arithmetic functions, $+$ $-$ $*$ $/$, and a terminal representing a double-precision floating point ephemeral random constant in the range $[0, 1]$. Thus all we assumed about the pricing function is that it was an arithmetic function of the bid and ask.

Individual mechanisms were compared according to the criteria represented by F in order to judge their fitness during the evolutionary process. As in Section 4.1, market outcomes for each pricing rule were computed by simulating agents equipped with the Roth-Erev learning algorithm. We used the same numbers of buyers, 30, and sellers, 30, and 100 auction rounds, but with only 100 different supply and demand schedules, constructed by assigning agents different private values, drawn randomly from a uniform distribution in the range $[30, 1000]$, to evaluate each generation of each population of pricing rules. We ran less rounds than in the landscape experiment because, as is usual for evolutionary methods, we had to use many generations and large populations—running each of these for 10,000 supply and demand schedules would have taken a prohibitive amount of time.

⁵<http://www.cs.umd.edu/projects/plus/ec/ecj/>

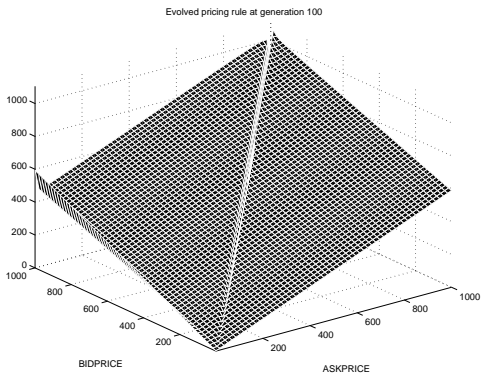


Figure 7: The transaction price set by the evolved auction rule.

Figure 10 shows part of the actual pricing rule that was evolved after 90 generations. This has been algebraically simplified, but as can be seen it is still far from straightforward, something that is not surprising given the way that standard genetic programming approaches handle the evolution of a program. Plotting the surface of the transaction price as a function of p_b and p_a , given in Figure 7, and comparing it with the surface for:

$$0.5p_a + 0.5p_b$$

(given in Figure 8) shows that these two functions are approximately equal apart from a slight variation when the ask price is very small or when the ask price is equal to the bid price. Thus the experiment effectively evolved a pricing rule for a discriminatory-price k double auction with $k = 0.5$ from the space of all arithmetic functions of ask and bid price.

Although the fitness landscape for this benchmark problem is very simple, we see this as a means of validating our design technique before we move on to more complex scenarios. Future work will investigate the use of this technique for more complex market scenarios, and will include other aspects of the auction design in the search space: for example, matching rules, bid validation rules and so on. Future work will analyse the evolved rules for a number of different market scenarios, for example where we have many more buyers than sellers and visa versa.

5. DISCUSSION

Parameter	value
Population size	4000
Selection	Parsimony Binary Tournament
Cross-over probability	0.9
Reproduction probability	0.1
Parsimony size probability	0.005
Cross-over maximum tree depth	17
Grow maximum tree depth	5
Grow minimum tree depth	5

Table 1: Koza GP parameters

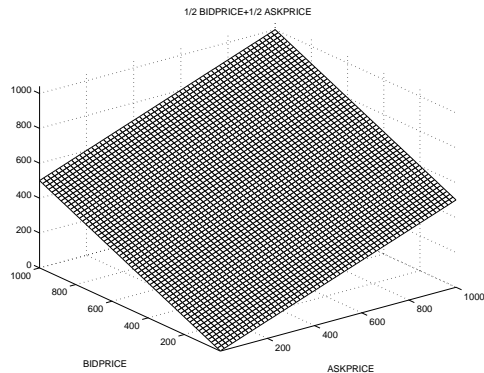


Figure 8: The transaction price set by the rule $0.5p_a + 0.5p_b$.

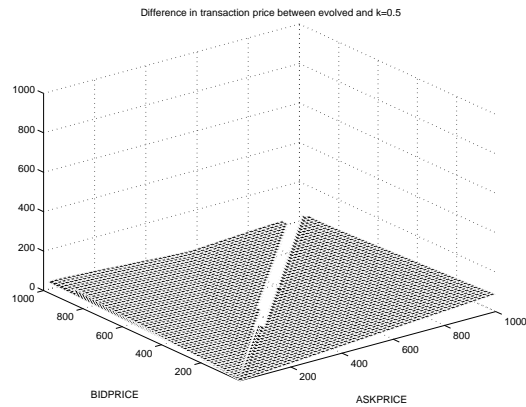


Figure 9: The difference in transaction prices between the $k = 0.5$ rule and the evolved rule

These results suggest that the approach we are adopting is a reasonable one—we have managed to evolve a rule which not only provides a high fitness, but also generates a rule that, in terms of the prices it sets, is close to a well established rule from the economics literature. The results also support the existing k -double auction rule since our GP search through the space of all functions of the bid and ask price has converged on a version of the k -double auction rule. This is in contrast to the results obtained by Cliff [3, 4], which discovered a new form of auction between classical buy-side and sell-side auctions.

Interestingly, this result also sheds some light on a problem we encountered with the approach we adopted in [26] when we used genetic programming for both evolving auction rules and evolving trading strategies. In those experiments we noticed that k -double auction pricing rules were evolved early on, when the strategies used by the traders were poor, but did not thrive. It seems it is possible that k -double auction rules do well provided that they are used in auctions with fairly good traders—in auctions with poor traders other rules, which are incompatible with good traders, do better.

This is consistent with a recent view proposed by Philip Mirowski [22, pp. 536–545] of economic marketplaces as complex ecologies. Some markets, such as garage sales, have relatively simple rules and procedures, while others, such as

$$\begin{aligned}
& ((0.6250385(0.93977016(ASKPRICE + 0.76238054))) + ((((-0.19079465)/(ASKPRICE - (((BIDPRICE \\
& + BIDPRICE)/((((ASKPRICE - 1) + 1.6088724)/(((1 - ASKPRICE) - (ASKPRICE/ASKPRICE)) + \\
& (2.5486426 + (BIDPRICE + 0.000012302072)))))) + ((BIDPRICE/ASKPRICE) + ((BIDPRICE + BIDPRICE) \\
& + (1.430315)/(BIDPRICE \cdot ASKPRICE))))))ASKPRICE)) \dots
\end{aligned}$$

Figure 10: The first few terms of the derived pricing rule.

financial futures markets, are, by comparison, very complex. Yet all manage to co-exist, with each type of market, apparently, finding its own niche in which to survive and prosper. Indeed, the oldest markets have survived for hundreds of years without rules from the newer ones being adopted in them. The behaviours of the participants in the different markets are, as one would expect, different. One challenge for computational economics, says Mirowski, is to explain this diversity, how it has arisen and how it is maintained.

Finally, we acknowledge that the solution concept we employ is based on a model of learning, and that this is not necessarily the most plausible solution concept for the double auction, an institution which is well known and in which traders are more likely to play analytically-derived strategies handed down by experts, rather than learn their strategies from scratch by trial and error. Our future work on auction mechanism design will employ a similar approach to that of [34], in which agents learn to converge on mixes of *high-level* strategies developed by experts—such as Cliff’s ZIP traders [5], Preist and van Tol’s PS traders [28], or traders using Gjerstad and Dickhaut’s mechanism [13]—rather than mixes of low-level strategies corresponding to fixed markups, which we assume in our current work. In such a scenario, the output from the Roth-Erev learning algorithm would be used to select which high-level strategy to use, rather than selecting which markup to use, and the asymptotic outcome would tell us which mixes of high-level strategies are stable, and thus likely to be adopted in the long term.

However, our long-term goal is the application of evolutionary mechanism design to design problems such as the congestion game [11], in which the payoff matrix changes on each play of the game, making the use of expert high-level strategies impractical; in the congestion game agents have no choice apart from to learn their strategies from scratch. In such a context, models of game playing based on learning, such as the Roth-Erev model, will play a vital role.

6. CONCLUSION

In this paper we have reported the results of two experiments in which we have examined the use of different pricing rules in a discriminatory price double auction. In part of this work we used adaptive buyers and seller agents to evaluate the effect of changing the parameter k in the k -double auction pricing rule. The second part of the work then successfully used genetic programming to automatically acquire a transaction-pricing rule. The method we describe here could be used to automatically generate a discriminatory pricing rule for a continuous double auction that meets any specific socio-economic requirements.

The work described here is part of a larger research effort aimed at creating techniques and methodologies for *computer-aided auction design*. We have thus far identified two promising techniques that could play a part in such a

technology: co-evolutionary mechanism design [26], and the optimisation technique described in this paper. These approaches are not mutually exclusive; we envisage that they will complement each other, and indeed complement standard analytic approaches to auction mechanism design.

For example, we could use the optimisation approach to find a set of promising mechanisms that perform well when agents play adaptive strategies. The auction designer might then pick a few of these designs that look as if they meet the criteria in hand, and then subject them to standard game-theoretic analysis, thus using the optimisation technique as a method of reducing the search-space for manual analysis. Once a mechanism has passed equilibria criteria, it might then be subjected to co-evolutionary experiments to probe it for “non-strategic” weaknesses in the protocol.⁶

When doing this the “prey” population would be pre-populated with our candidate mechanism, and the “predator” population would be pre-populated with equilibrium bidding strategies; the predator population might then find non-strategic weaknesses in the auction population thus driving it to more robust areas of the design space. This whole process of:

1. identify promising mechanisms through search;
2. pick and solve for equilibrium solutions; and
3. use co-evolutionary learning to identify non-strategic weaknesses,

might then be iterated through until no further weaknesses are discovered. At the end of the design process we hope to have auction mechanisms that: perform well against adaptive, possibly non-equilibrium, strategies; that perform well in equilibrium; and are robust against non-strategic predatory behaviour.

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⁶By a non-strategic weakness we mean a weakness in the mechanism that occurs when game-theoretic assumptions are violated due to unforeseen real-world circumstances. For example, auction theory assumes that agents are not able to signal to each other in order to collude, but the theory cannot account for features of the auction protocol that enable agents to use the auction protocol *itself* to send covert signals. It was just such a weakness in the German radio-spectrum auctions that seems to have allowed Manesman and T-Mobile to use the low-order digits of their bids to signal to each other and form a collusive price-fixing strategy[17].

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