MACHINE INTELLIGENCE RESEARCH UNIT

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ABSTRACT

Experiments were conducted with Class A chess players for both the Rook's side and Knight's side of the KRKN ending to learn how hard correct play is. New drawn positions were discovered where the defending king and knight are separated. As a result some additional heuristics have been developed. Further experience with strong master players is reported.

Notation

Symbol	Meaning
!	A very good move
!!	A spectacularly good move
?	A bad move which changes a won position into a draw or a drawn into a lost position
?!	A bad move inspired by a good idea
+	Check
+-	White wins
=	The position is drawn
K	King
R	Rook
N	Knight
Q	Queen
P	Pawn
B	Bishop

Advice Language 1 (AL1: Michie, 1976) has provided a vehicle for translating human chess endgame expertise into machine-usable form. The endgame is the phase of chess where it is believed the greatest subtlety and domain-specific knowledge is necessary. An endgame which has received attention is King and Rook vs. King and Knight(KRKN). Both won and drawn positions exist, there being a total of approximately three million legal positions.

Reports on implementations in Advice Language appear elsewhere (Bratko, Kopec & Michie, 1978; Bratko and Michie, 1979). This note discusses the chess in KRKN.

The basic text used for the purposes of our research was the 8 pages in Fine's, Basic Chess Endings (BCE). Bratko had also used the following position, Diag. CORNCASE:WK:a3,BK:a1,WR:b8,BN;c1, from Keres' Practical Chess Endings (PCE) as a special case he called Corncase. Black to move can only draw with 1....Ne2. On 1. ...Na2 2.Kb3,Kb1 (2....Nc1+ 3.Kc2+-) 3.Rb7!, Nc1+ 4.Kc3+,Ka1 or Ka2 and 5.Kc2 wins, or if 1. ...Nd3 2.Kb3,Nc5+ (2. ...Nc1+ 3.Kc2 +-) 3.Kc4, Ne4 4.Rd8,Kb2 5.Rd4 etc. wins by a technique which will be exemplified a number of times in this report.

In 1977, an experiment was conducted to learn about how hard the play for both sides in KRKN is. In this experiment, which we shall call Experiment 1, there were 3 subjects from the Edinburgh University Chess Club, averaging 1882 in rating. Each subject was assigned two positions; one being a drawn position which the subject was required to defend correctly for the N-side, and the other being a lost position where the subject had to find a winning sequence for the R-side.

Drawn positions were those used in testing the Bratko Advice Table (Bratko & Michie, 1979) and were labelled T1,T2,T3 (T1=Corncase, T2 = WK:e6,WR:a7,BK:f8,BN:g8, T3 = WK:c3,WR:e2,BK:e4,BN:f3). Won positions were carefully chosen from BCE based on the stated longest number of moves required to win the longest being 24 moves in the main variation from Diagram No. 501 (original source Berger), (WK:c6,WR:h8,BK:a7,BN:b7, B to move). The second and third longest were stated as 17 moves (WK:c6,WR:d8,BK:a7,BN:e3, W to move) and 15 moves (WK:d6,WR:d2,BK:b6, BN:f6, W to move). However, the longevity of win in these positions was

not substantiated when verified with a database (See Kopec & Niblett, 1979). All positions were ranked in difficulty and subjects were assigned test positions so that there was an overall balance in difficulty amongst them. Of course length does not necessarily correlate with difficulty, but this method of ranking was chosen for its convenience.

The experiments required that the subjects complete their tasks in 1 hour. That is, they were first to defend KNKR over 30 moves (resignation, loss on time, checkmate, or loss of the N being termination conditions) against the author, who is a U.S. National Master, and then after a short break, they were to try and win their allotted won positions in 30 moves against the same opponent. I also had 1 hour for 30 moves. The subjects were requested to state any specific knowledge they had about KRKN or KNKR (won positions or drawn positions respectively). The experimental sessions averaged about 3 hours.

There were several interesting discoveries for us from this experiment. Subjects seemed to have no trouble defending the drawn positions which the Bratko Table had been tested on. Yet, on going over Corncase with one subject, Philip Edwards (1840), after the experiment, I found that he did not seem to know why the other N moves lost and had chosen 1. ...Ne2 "intuitively" (see Appendix A). This was surprising because 1. ...Ne2 would seem to be the counter-intuitive move, separating BK and BN. In winning won positions the subjects went wrong in 2 of the 3 test cases. However, the one subject who did win, Pat Coleman, did so in 11 moves, 6 less than the variation given by Fine (see Appendix B). A later confirmation with the Thompson KRKN database (a computer generated complete lookup table from which minimax-optimal play for both sides can be ascertained) indicated that the initial position (WK:c6,WR:d8,BK:a7,BN:e3 (No. 499 BCE)) is in fact won in 10 moves with best play by both sides. The retrospective analysis of subject Philip Edward's play in failing to win proved to be most rewarding. From the position, WK:d6,WR:d2,BK:b6,BN:f6 (15 moves in BCE) W: Edwards B:Kopec, play went:

1. Rb2+ Ka5

2. Kc6 ?!   White indicates he has some ideas about how he
                   might win, but the correct move was 2.Re2 with
                   a sure win, i.e. 2. ...Kb5 3.Re5+!,Kc4 4.Re6!
                   Nh5 5.Rh6 (Not 5.Re4+?,Kd3 6.Rh4,Nf6 7.Ke5
                   Nd7 and it does not seem W can make any

further progress), Ng3 (If 5. ...Ng7
6.Ke5 wins quickly) 6.Rh4+!, Kb5
7.Kd5,Ne2 (If 7. ...Nf5 8.Re4 etc.)
8.Rc4,Ng3 9.Rc2,Nf5 10.Re2,Kb4
11.Re5, Ng3 12.Kd4 etc. and it is clear
the N will be picked up in a few moves.
This maneuvering, taking advantage of pins
and forks, is a typical method by which
White wins when the BK and BN get consider-
ably separated. Just what is "considerable
separation" is a difficult question. For
Bratko's Advice Table, separation was defined
as when the BK and BN are 4 or more K moves
apart. Of course there are problems in such
an absolute definition since the relative
position of the other pieces will affect
the relations between the BK and BN.

The record of the Edwards' sequence continued:

2. ... Ne4   (Main Diagram)

3. Rb1 Nc3

4. Rb3 Na4   Now a standard position which we call the "Edge
                     Draw" has been reached. This type of drawn
                     position in KNKR was known 300 years ago (Lolli:
                     Roycroft, 1972).

5. Rb1 Nc3

6. Rb8 etc. and White offered a draw after move 22. In many positions
of the Edge Draw Black can only play one move, but for this reason
the draw is even clearer.

Returning to 2. ...Ne4 (Main Diagram) I was interested
in the value of the position. The study of this one position sprouted
a whole family of positions which indicated the richness of the ending,
even though only 4 pieces are involved. The first move tried was 3. Rb3.
Then on 3. ...Ka4? 4.Rd3 Black is in a bad way (4. ...Nf2 5.Rd2) and
his stranded knight will soon be captured. Correct was 3. ...Nd2 (Diag.
D1) when after 4.Rd3,Nb1 (Not Nc4 5.Kc5+-) draws. Then I decided to
investigate an even stronger looking first move from the Main Diagram --
1.Kd5,Nc3+ 2.Kc4,Ne4 (Not Na4 3.Ra2) and now there is only one winning
move which you would select only after performing some considerable

will be discussed fully later. Correct is 3.Re2!, Nd6+ 4.Kc5,
Nb7+ 5.Kc6,Nd8+ (5... Ka6 6.Ra2+,Na5+ 7.Kc5 +-) and White soon wins.

Hence from the Main Diagram is White winning after
1.Kd5! ? No, the stunning, counter-intuititive 1.... Nf6+!! draws!
The answers to White's 7 possible K moves are given in counter-clockwise
order: (a)Ke6,Ne4 (b)Kd6,Ne4+ (c) Kc6,Ne4 (returning to Main Diag.)
(d)Kc5,Nd7+ or Ne4+ (e)Kc4,Nd7 (f)Kd4,Nd7 (g)Ke5,Nd7+. 1. ...Nf6+!!
is the first case from our experimental work where a counter-intuitive
"separating move" is required.

Now turning to Diagram D4, Black has only one
drawing move, for if 1. ...Kb6 2.Rd4,Ng5 (On 2. ...Ng3 3.Kd5,Nf5*
4.Re4,Kc7 5.Re5,Nh6 6.Ke6 etc.) 3.Rg4! continuing:
i. 3. ...Nf7 4.Rg6+,Ka5(forced) 5.Kd5! and White wins in 4 moves
(database) or
ii. 3...Nf3 4.Rg6+,Ka5 5.Kd5 (Now if 5. ...Nd2 6.Rg2,Nb3 7.Kc4! wins),
Kb4 6.Rg4+,Kb5 7.Rg2,Kb4 8.Ke4 wins.
Going back to D4, only 1. ...Kb7 draws. After this move it is fairly
clear that White cannot prevent the reunion of the BK and BN via
Nf6-Ne8-Nc7, Nf6-Kc7-Nd7, Kc6 (c7)-Nd6 or Nf6-Nd7. I have defined these
meeting squares as "Common Ground", and White's inability to prevent
Black's achievement of Common Ground guarantees a draw. Diagram 3
depicts the possibilities for Common Ground after 1. ...Kb7 from D4.
Bratko's Table played the phlegmatic 1. ...Ka7 after which only the
problematic 2.Rd7+! wins. Now if 3. ...Ka6 or 3. ...Kb6 we transpose
into the variations given above for 1. ...Kb6, so only 3. ...Kb8 needs
to be considered seriously (...Ka8 is pointless). Thus 4.Rd4, Nf6
5.Rd6, (our investigation of the perfect play offered by the database
has taught us that such direct attacking moves by the R on the fleeing
N are frequently very strong) Ne4 6.Re6,Ng5 7.Rg6,Ne4 8.Kd4 wins quickly.

Thus this retrospective analysis stemming from
the Main Diagram, has brought out a fact which Bratko and his sources
(Fine's BCE and Keres' PCE) had overlooked: namely that there is a whole
domain of positions which lie on the boundary of the win-draw territory
for KRKN which may entail a somewhat counter-intuitive,separating N move
or a subtle K move to avoid forthcoming tactical losses. For example
from D4, the move 1. ...Kb6 brings out a conceptual rule which helps
to curtail the amount of "blind" look-ahead necessary, which I formulate:
If K and N are 3 or more K moves apart, don't move your K onto a file

With these ideas in mind, I was able to generate quite a
number of positions where the original Bratko Table would fail to
play the correct defensive move(s) and this was not surprising
since such positions had previously been considered lost. Some ex-
amples of these positions follow. In Diag D6, only 1. ...Kc8 draws
by ensuring Common Ground. If 1. ...Nf6 2.Rd6 wins quickly. The
same motif arises in D15 where 1. ...Kd6 draws. If instead 1. ...
Ng3? 2.Re3 wins, but 1. ...Nh2 also draws. Also in D17, only 1. ...
Kg3 draws. After 1. ...Nb5? 2.Rh3! wins.

When given a choice, the best path for the N back to its K --
the central route or the peripheral one -- depends on the precise
positioning and interactions between the 4 pieces. For example,
in D9 after 1. ...Nc6+ (Not 1. ...Nf5+?2.Ke5 etc. against which the
heuristic could be: "Don't move your N towards both the opposing K
and R unless its the only way back to your K.") 2.Kc4 leaving
Black with more ways to go wrong than 2.Kc5. Now only the peripheral
2. ...Na5+ draws, while the central 2. ...Ne5+? loses similarly to
1. ...Nf5+ after 3.Kd5. On the other hand in D10, the peripheral
1. ...Na4? loses after 2.Kb4 since 2. ...Nc3 is impossible, but
1. ...Ne4 2.Kd4,Nd2! just draws; or if 2.Kd3,Nc5+ and Black can repeat.

I conclude this section with a position contrived to demonstrate
that distance between the BK and BN is not the fundamental measure,
but rather the number of paths between them.

The investigation began with Diag. T (for trial) (WK:b3, WR:h8,
BK:d1,BN:a7) where I wanted to show: 1) The BK and BN can be quite far
apart and the position is still drawn. 2) The path chosen by the N must
offer options for Common Ground (usually central) rather than necessarily
minimize K and N distance. That is 1. ... Nc6 over 1. ...Nb5. In fact,
I discovered the position is lost after all! On 1. ...Nb5 2.Rd8+,Ke2
3.Kb4! (not 3.Rd7,Ke3=), Nc7 4.Rd7 wins handily, while 1. ...Nc6 also
loses to 2.Kc3!, Ne5 (If 2. ...Ke2 3.Re8+,Kf3 4.Re6,Nd8 5.Rd6,Nb7
6.Rd5,Kf4 7.Kb4! snatches the N) 3.Rd8+,Kc1 4.Re8,Nf3 5.Re3 and wins.
Since the mating threats utilizing the positions of the K's play a vital
role in the above variations. I tried only altering the position of the
BK. The first such position found where 1. ...Nc6 draws and 1. ...Nb5 loses)
was with the BK on f1. (Diag D9). Here after 1. ...Nc6 it is fairly
clear that White cannot cut off both of Black's threatened paths to

Common Ground (Ne5 and Nd4). But after 1. ...Nb5 2.Rd8 (Diag D8.2)
(the normal winning try, preventing both ...Nd6 and Nd4), Nc7!! is
White winning? International Master Craig Pritchett and I pondered
over this position for a half hour, being unable to find a win. In
fact the position is drawn! After further consideration from White's
point of view, 2. ...Nc7 makes more sense; Rd7 winning the N in a few
moves was threatened. The N moves directly away from its K into the
hinterlands of our 8x8 territory, and yet it can persist in fleeing
from the enemy K and R. A continuation might go: 3.Rd6,Kf2 (Not 3. ...
Ke2 4.Rd7) 4.Rf6+! (On 4.Kc4,Kf3 5.Kc5,Kf4 (or Ne8) Black draws without
much difficulty), (Diag. D8.4) 4. ...Kg3! (not 4. ...Ke3 5.Kc4,Ke4
6.Rc6! +-). Now after 5.Kc4 we reach Diag. D18. Black's drawing move
is 5. ...Kh4!! It is the only move which can handle the threatened
1.Rc6,Ne8 2.Re6,Nc7 3.Re7 etc. i.e. 2. ...Ng7 is now possible. The
logic of this move may be stated as a heuristic which is an inductive
step on one mentioned earlier: "Don't move your K onto a file or rank
the N may need to use in order to be united with the K."

This concludes my survey of drawn positions in KRKN. They indicate
that further investigation into the results of one aspect of Experiment
1 was necessary: "Are Class A players able to handle the defense of
KNKR?"

For this purpose Experiment II was designed. The experimental
conditions were similar, except that this time subjects were required to
hold 3 drawn starting positions for 20 moves, with 40 minutes for each.
Of the 16 benchmark positions which had been accumulated, the original
Bratko Table had failed on 13 of them. Study of the concepts involved
in these 16 positions facilitated their ranking for difficulty. Since
most "A" players will know that they should keep K and N together,
positions where counter-intuitive separating moves are necessary were
considered most difficult. Of the 9 positions chosen for the Experiment,
the Bratko Table had failed on 6 of them. The average rating of the
subjects was 1947. In 5 of the 9 test treatments subjects made errors
despite playing the correct first move (considered most difficult).
Two of three subjects knew at least that their K and N should be kept
together whenever possible.

The results of this experiment gave evidence that at least expert
and perhaps master strength may be required to defend KNKR correctly.
Some specific concepts, discrete pieces of knowledge, involved in the
correct defense of separated K and N positions have been defined.

An important aid in the verification of the value of these positions
has been the Ken Thompson database. Thompson has written programs which
generate databases for all 4-piece endings. Databases also exist for
KRP vs. KR, KQP vs. KQ, KRK, KPKP (pawns on same file), and KPK. Readers
are referred to Clarke (1977), Kopec & Niblett (1979), and Arlazarov
& Futer (1979) for further information on the construction and organ-
ization of databases.

One of the two longest wins of 27 moves found by the database
follows: (Diag. L, WK:d1,WR:h1,BK:b1,BN:g4) Along the way there may be
more than one move, after the first, which is optimal for White. However
there are no "improvements" for Black in the sense of variations which lose
in more than 27 moves, only less. Readers are invited to do some veri-
fication for themselves and see how difficult it is even to keep track of
the distinct variations. Note also how the N must wander far afield and
then be permitted to reunite with the BK in this, the best variation.

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 1. Rh4    Ne5
 2. Re4    Nf7
 3. Rb4+   Ka2
 4. Kc2    Ka3
 5. Kc3    Nd6
 6. Rb6    Ne4+
 7. Kd3    Nf2+
 8. Kc4    Nd1
 9. Rb3+   Ka4    Here if 9. ...Ka2 10.Rb8,Nb2+ 11.Kc3 transposes into the
                  well-analysed al-Adli position (Averbakh, 1979) whose
                  background will be more fully described later.

10. Rf3    Nb2+
11. Kc3    Ka3
12. Rg3           A pretty passing move.
12....     Na4+
13. Kc4    Ka2
14. Kb4    Nb2    A form of the "al-Adli" position has been achieved anyway.
15. Rg4    Nd3+
16. Kc3    Nc5
17. Rc4    Ne6
18. Ra4+   Kb1
19. Ra5    Ng7
20. Re5           The rest involves just picking up the trapped N.
20....     Ka2
21. Kd4    Kb3
22. Kd5    Kc3
23. Kc6    Kd4
24. Kd6    Kd3
25. Ke7    Kd4    26. Rg5 etc.
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At this point it seems appropriate to mention a little of the long
history of KRKN, which dates back to the ninth century A.D. when it was
analysed for the ancient game of Chaturanga. The only pieces whose
movements are the same as for chess are the Rook and Knight. At that

time an Arab, al-Adli, analysed the diagrammed position (Diag. AL, WK:b5,WR:h8,BN:b7:a7) correctly as a win. Quoting from Averbakh (1978,p107), "In 1899, Berger, in his first edition of <u>Theorie und Praxis der Endspiele</u> subjected the position to a detailed analysis, which however, contained a number of mistakes." (See Averbakh for a more complete account).

Fine and others have used this faulty analysis, as the following shows:

1.Rf8!	(Fine's punctuation) Rh5 is best (database) winning in 13 rather than 14 moves. Here Fine's notes say, "Not Rh7 because of ...Kb8 2.Kb6 drawn." However 2.Kc6 wins, and more quickly than in the main variation given by Fine. This is one of two cases where Fine gives the wrong game-theoretic value for a position. (Kopec & Niblett, 1979)
	What Fine probably overlooked is that on 2.Kc6!,Nd8+ (If 2...Na5+ 3.Kb6,Nc4+ 4.Kb5!,Nd6+ 5.Kc6 etc.) 3. Kd7,Nb7 (all forced) 4.Rh5, Ka7 5.Kc8!! (Not 5.Kc6, Ka6 6.Rh6!,Na5+ 7.Kc5+,Ka7 8.Kb5 etc. cycling) wins in a few moves.
1. ... Nd6+	forced.
2. Kc6 Nc4	
3. Rd8!	In spite of Fine's exclamation mark, this move changes the depth of win from 11 to 17 moves in this position (Kopec & Niblett, 1979)

Another long KRKN win from the database (24 moves) which has been used to test humans on, is the position in Diag. R2 (WK:d6,BK:f7,WR:h5,BN:c3) White to play. A.J. Roycroft, a leading endgame analyst, (see Appendix C of Kopec & Niblett, 1979) was given this position along with two others having the same depth of win, and a week to analyse them (similar to completing an adjourned game). He was also given four other "24 movers" and their optimal solutions for training. Then he was asked to play each of the first three 24-movers against the Thompson database, using whatever notes he had, plus an auxiliary board on which he could move pieces around, and a rate of 16 moves per hour. He completed the tasks without an error in each case. Not long after this, I.M.Pritchett was given the position in Diag. R2 to play with tournament time conditions. Pritchett also knew that this entire "miniaturized version" of chess was not to be underestimated from my having shown him numerous positions and tricks. He found the same line which Roycroft had found with a week to study the position. In this case the defense was tackled by an updated version of Bratko's Advice Table. Unfortunately, the Table did not play the optimal defense on move 12. Yet Pritchett had played optimally in the first half of the task, which was

1. Ke5	Na4		13. Re3	Nd4
2. Rh7+	Ke8		14. Re4	Nb3
3. Kd6	Nb6		14. Kd6	Kg7
4. Rh8+	Kf7		16. Kd5	Nd2
5. Rh4	Nc8+		17. Rf4	Nb1
6. Kd7	Nb6+		18. Rf3	Kg6
7. Kc6	Nc8		19. Rd3	Kf5
8. Rh7+	Kf6		20. Kc5	Ke4
9. Rh6+	Kg7		21. Kc4	Ke5
10. Re6	Na7+		22. Kb4	Ke6
11. Kd6	Kf8		23. Rd1.	'resigns'
12. Kd7	Nb5			

The experiments with Roycroft and Pritchett show that, given the time
and some previous knowledge or experience, humans can find the challenge
of optimal play for the stronger side in KRKN positions within reach.

In the recent Lloyd's Bank Master's tournament (August, 1978), after
120 moves and three adjournments, the game Balinas-Benjamin had reached the
diagrammed position (Diag. 8B : WK:c2,WP:a3,WR:a1,BK:b4,BN:c5, W to play).
Grandmaster Balinas,   the exchange up for many moves, had now · --.
made the win difficult by cramping his Rook with the advance of the RP. He
came to the analysis room believing the position was now drawn. I showed him
the only win which seems to exist in the position: that is by reaching
al-Adli's position --- 1.a2,Ka3 2.Rh1, Kxa2 3.Kc3! Now if 3. ...Na4+
4.Kb4,Nb2 we have the basic al-Adli position, or if 3. ...Nb3 4.Rh5! wins
quickly, i.e. (a) 4. ...Na1 (the N in the corner always loses in these
positions) 5.Rh2+,Kb1 (or 5....Ka3 6.Rb2) 6.Rg2 +- (b) 4. ...Ka3 5.Rb5 +-
(c) 4. ...Nc1 5.Kc2,Nb3 6.Rd5,Na1+ 7.Kc3, Nb3 8.Rb5,Nc1 9.Kc2 +-. After
resumption of play Benjamin resigned on move 130.

Ivan Bratko's modified (August, 1978) KRKN Advice Table now plays the
correct move(s) in every one of our 20 benchmark positions (Kopec & Niblett,
1979). This is mainly due to a function, NSAFETY, which is based on the
mobility and multiplicity of paths to the BK for the N. Niblett, however,
has already found six more   "separated" positions where it errs (personal
communication). The investigation continues.

(W to move)   (W to move)
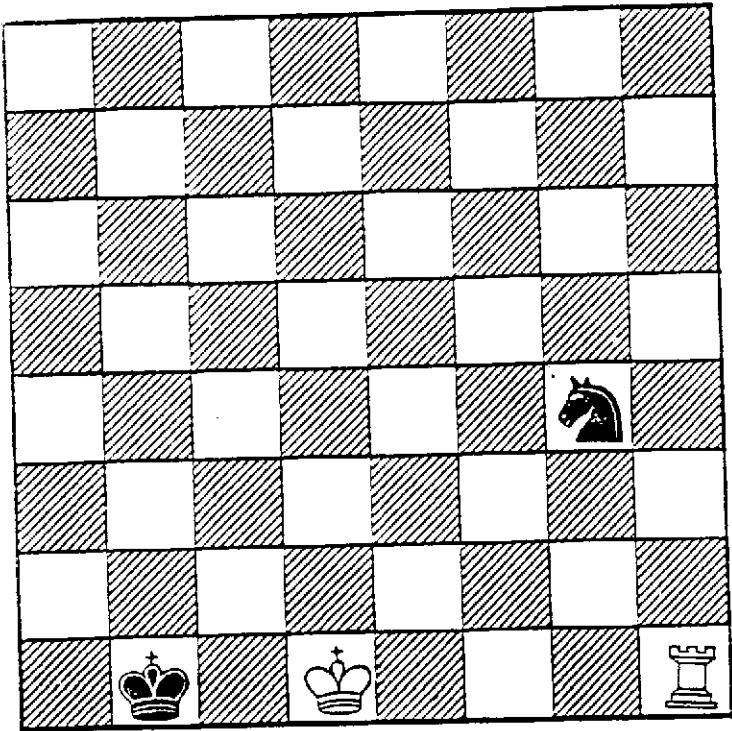
Diag. L   Diag. BB
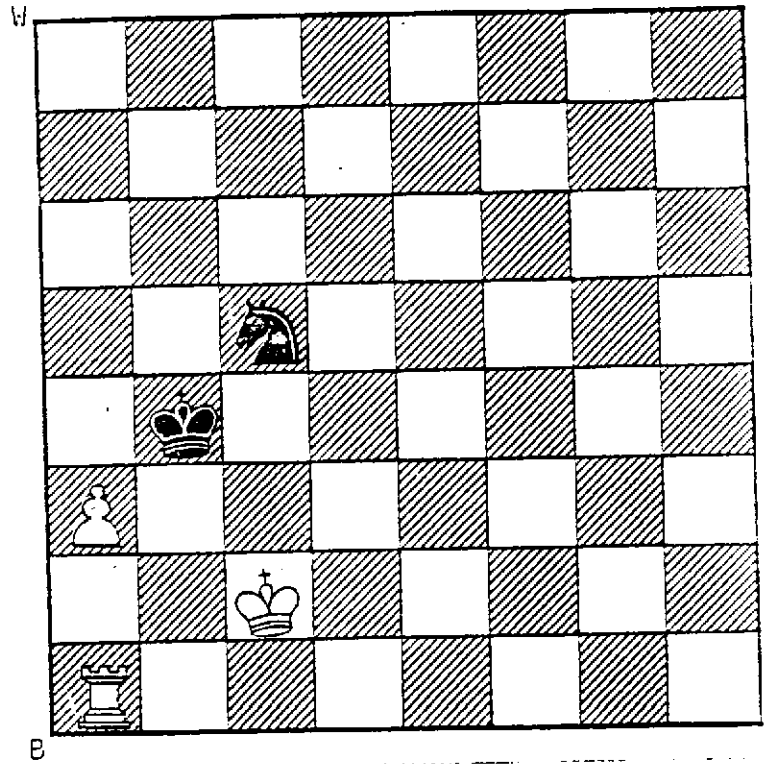
Experimental Record of subject Edward's play in Corncase

Date: October 19th, 1977

   Grading: 1840   Rate of Play: 30/1hr.   Experimenter: D. Kopec

   Starting Position: WK:QR3,WR:KN2,BK:QN8,BN:QB8 (See Keres, 1974)

White to Play

1.	R-N2Ch	K-R1		16. R-R1	N-Q7Ch	(32)
2.	R-N8	N-K7	(5)	17. K-B4	K-Q6	(34)
3.	K-N6	K-N8		18. R-R8	N-B5 ·	(36)
4.	R-QB8	N-Q5Ch	(11)	19. R-Q8Ch	K-K7	(39)
5.	K-B3	N-N4Ch	(14)	20. R-Q4	N-Q7	(41)
6.	K-N4	N-Q5	(16)	21. R-R4	K-Q6	
7.	R-B5	K-N2	(17.5)	22. R-R3Ch	K-Q5	(43)
8.	K-B4	N-K7	(19)	23. R-R8	N-N6	(46)
9.	K-N4	N-B5	(22)	24. R-Q8Ch	K-B6	(48)
10.	R-B5	N-Q6Ch	(25)	25. K-K5	K-B5	(50)
11.	K-B4	K-B7	(27)	26. R-B8Ch	K-Q6	(52)
12.	R-B1	K-Q7	(29)	27. R-B7	N-Q7	(53)
13.	R-KR1	N-K4Ch	(30)	28. K-Q5	K-K6	(54)
14.	K-Q4	N-B6Ch		29. R-B3Ch	K-B5	(55)
15.	K-K4	K-K7		30. R-Q3	N-B6	(56)

Subject had no previous knowledge of experimental domain

Note: Figures in "( )" are the cumulative time consumed by the
      subject.

W:P. Coleman B:D. Kopec	Elo Rating (1965) (2350)	Minimax-Optimal Value (Moves)	Optimal move(s) and their mini- max-optimal path-lengths
1. Rd4 (13)*		9	
1. ... Nf5		6	Kb8(9)
2. Ra4+ (15)		6	
2. ... Kb8		5	
3. Re4		5	
3. ... Ka7		3	Ng3(5)
4. Kd5		8	Kc7(3)
4. ... Kb6		8	Nb3,Kb7,Kb6
5. Ke6 (21)		7	
6. Re1		6	
6. ... Kc5		5	Kc7,Kc6(6)
7. Ke5 (22)		4	
7. ... Kc4		4	
8. Kf4		3	
8. ... Nh5+		3	
9. Kg5		2	
9. ... Ng3		2	
10. Kg4 (25)		and captures N	

*   Figures in "( )" after White's moves in this
    column are the total time from the allotted
    hour, consumed by subject.

This Appendix is taken from Kopec & Niblett (1979)
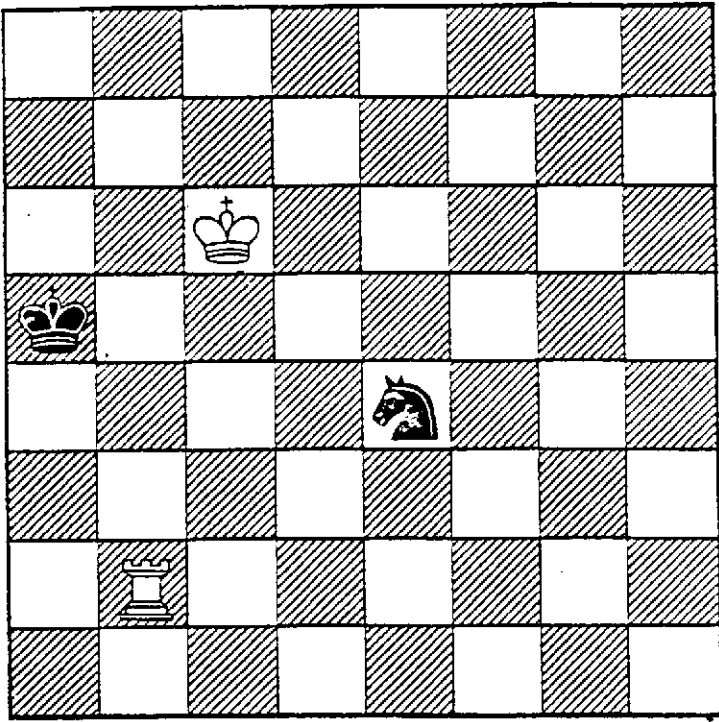
MAIN DIAGRAM
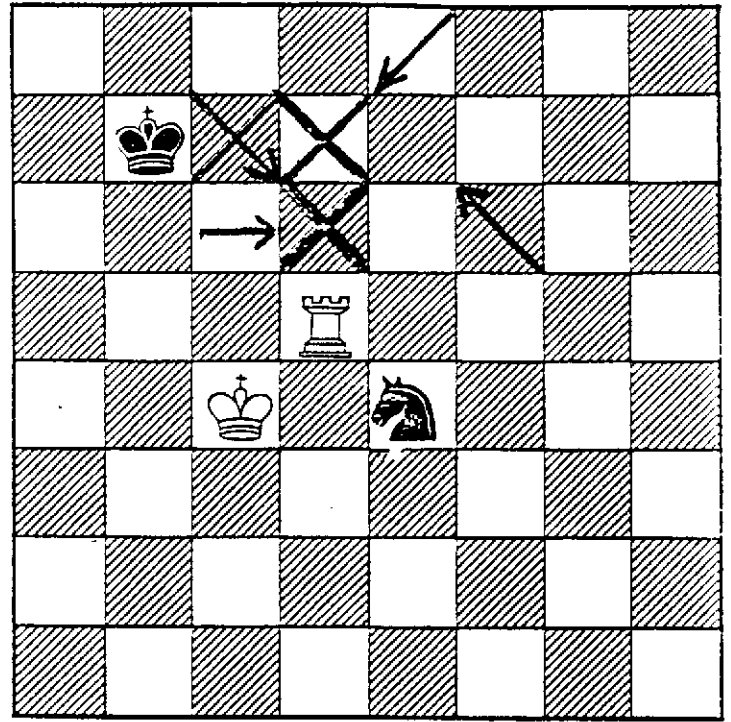
Diag. 3

Position after 1. ... Kb7 from D4.

Squares which are "Common Ground" or potential meeting squares for BK and BN are marked with an "X". Paths to these squares are marked with "➔" also indicating approximate direction to target square(s).
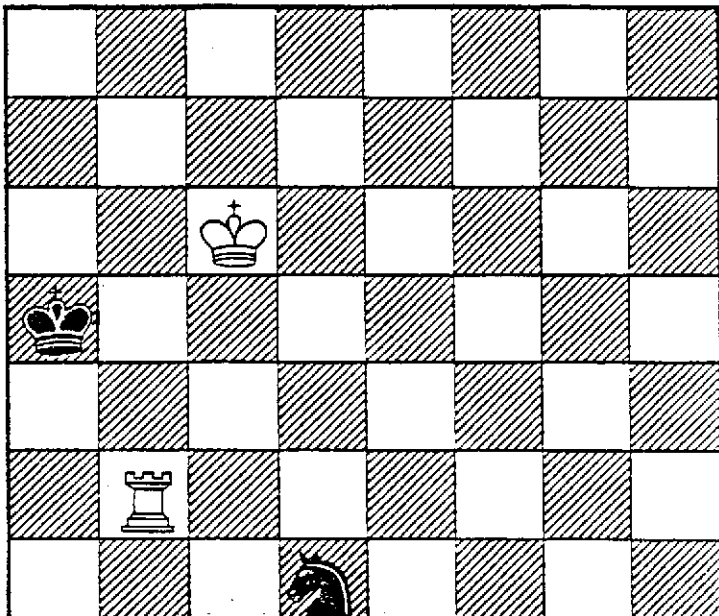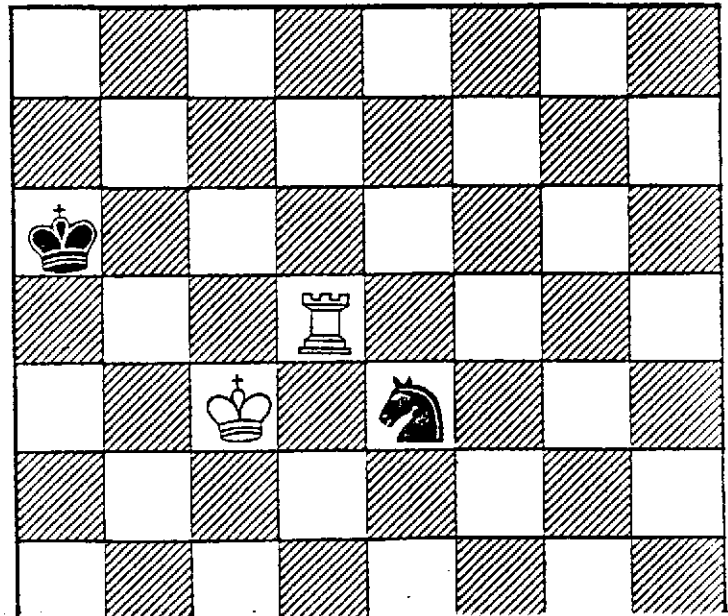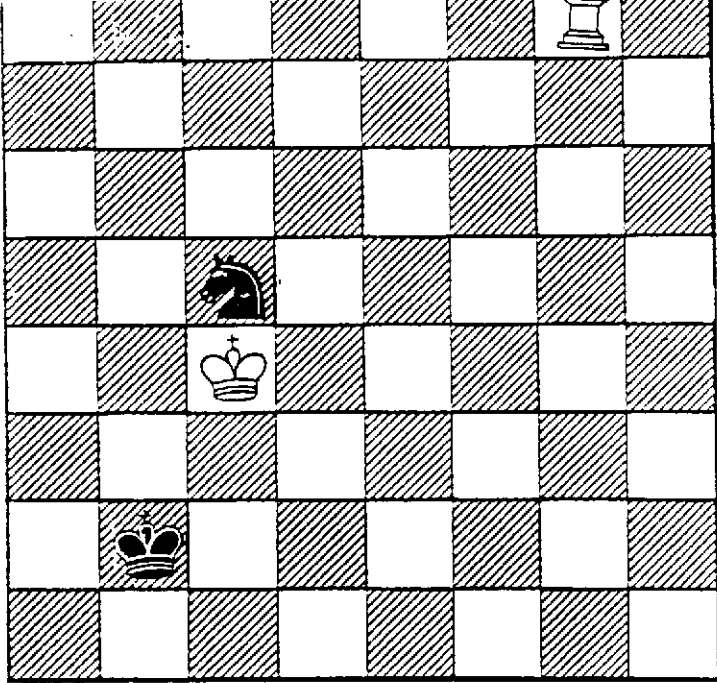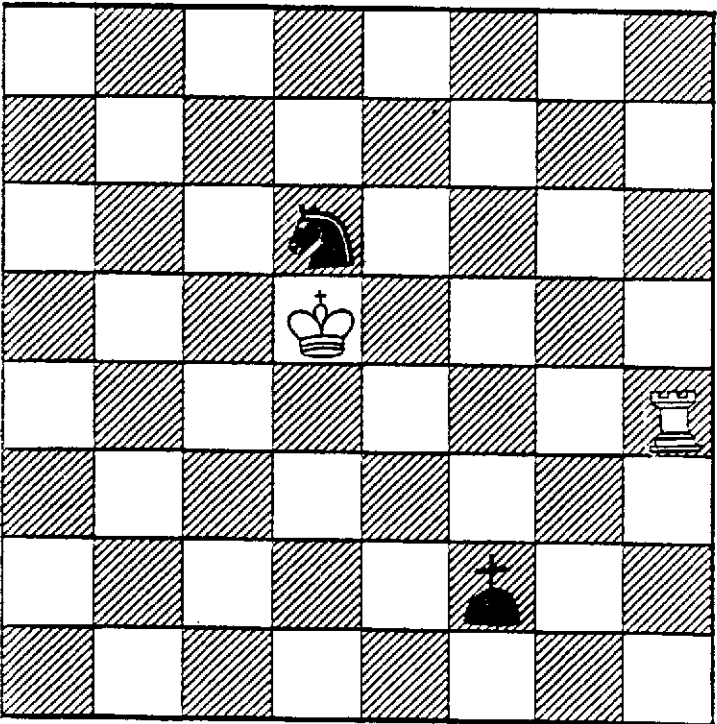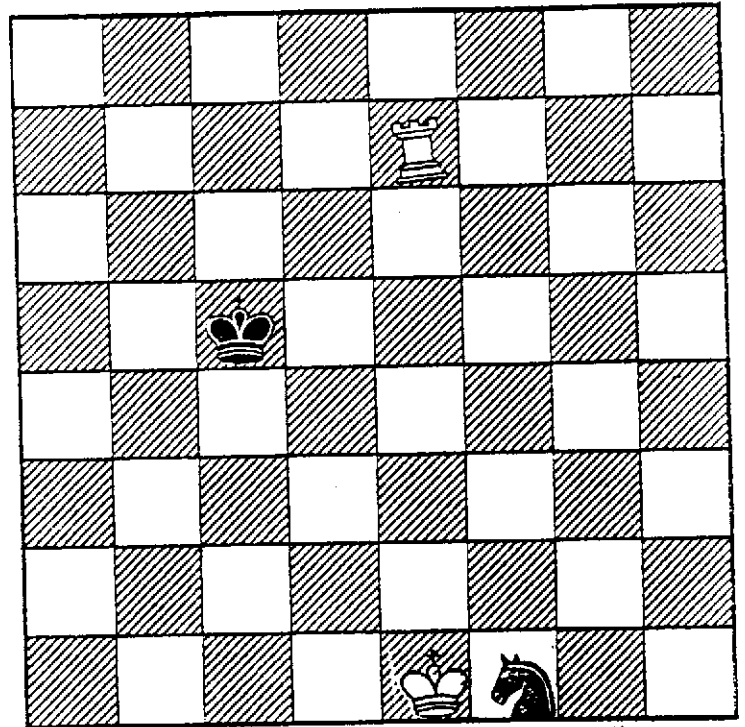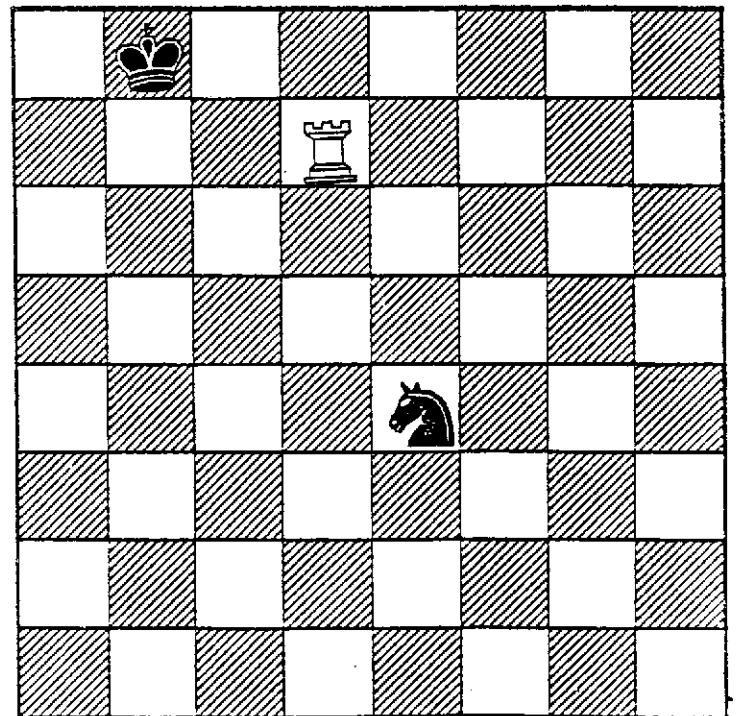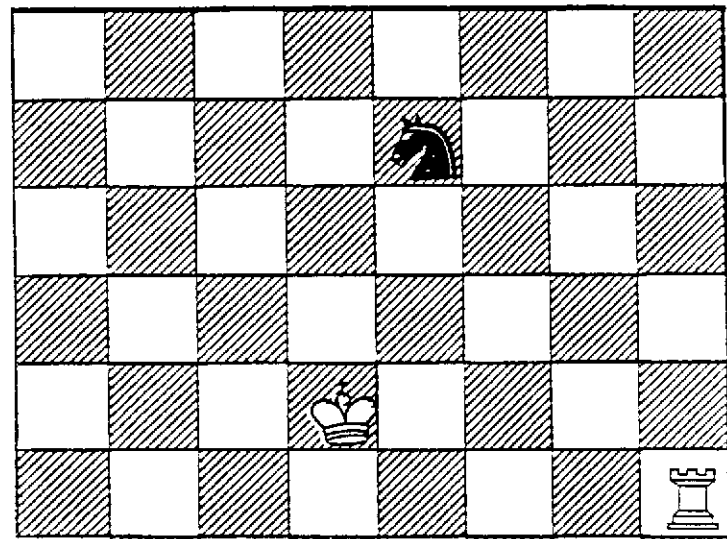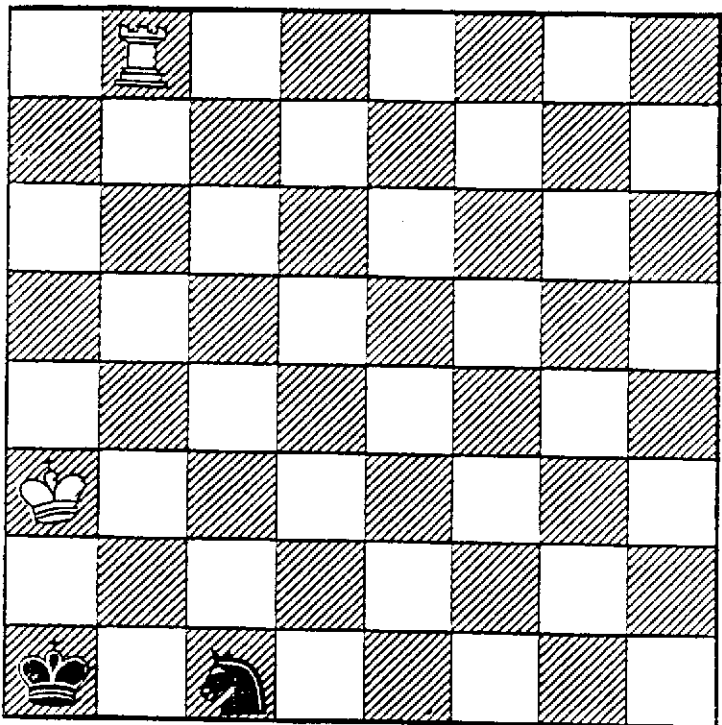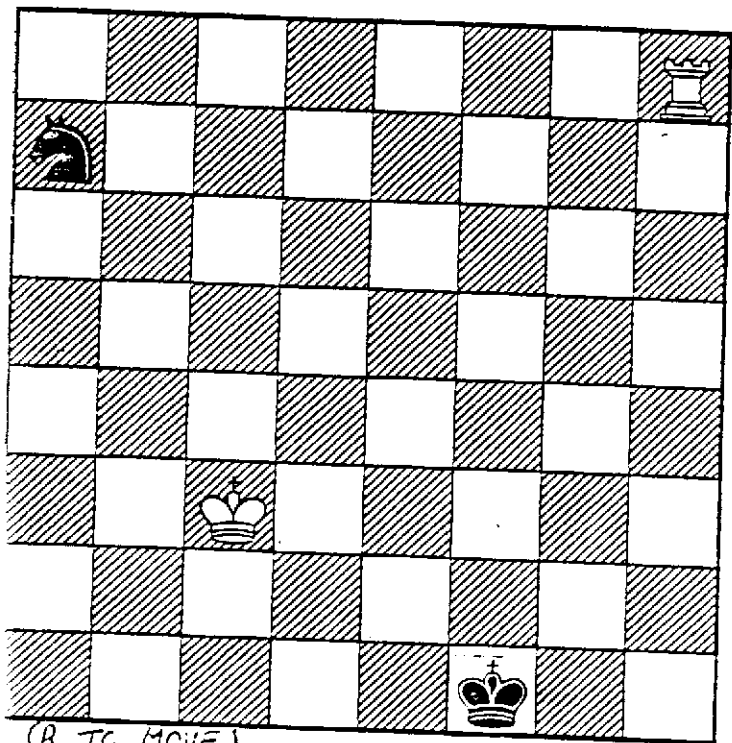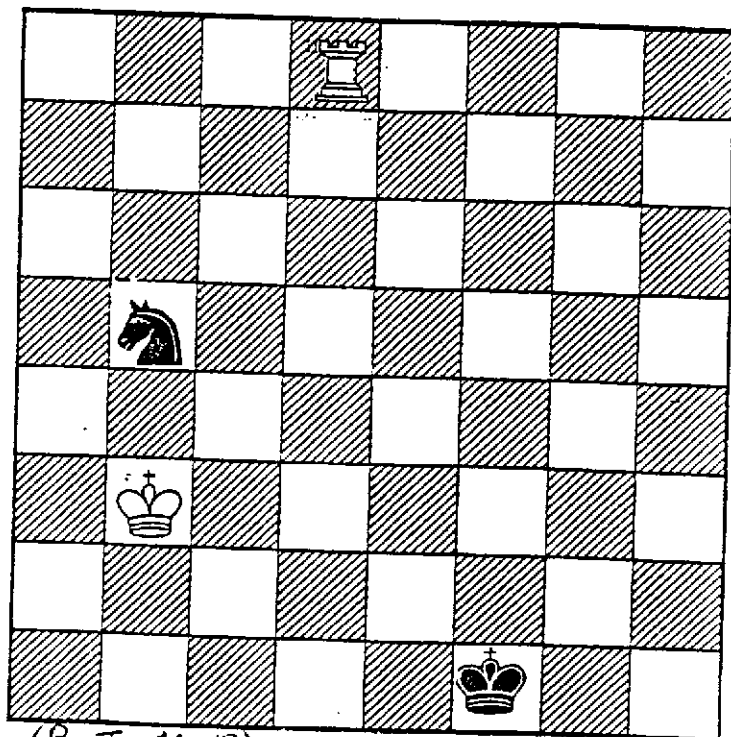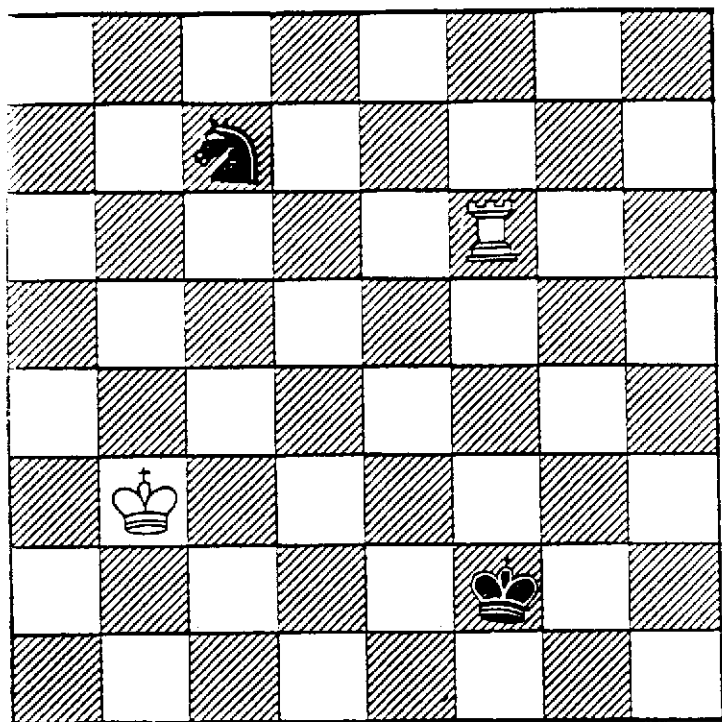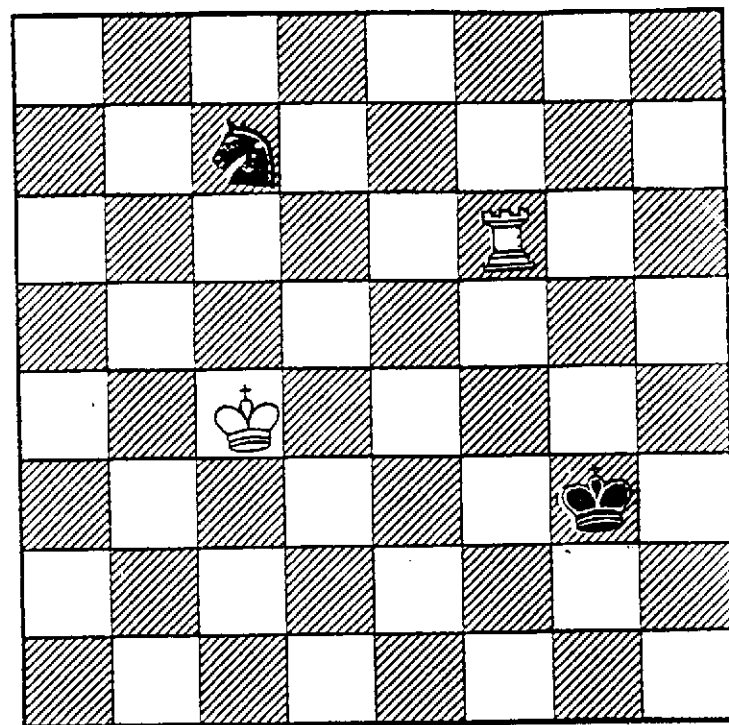
(B To Move)

Corncase

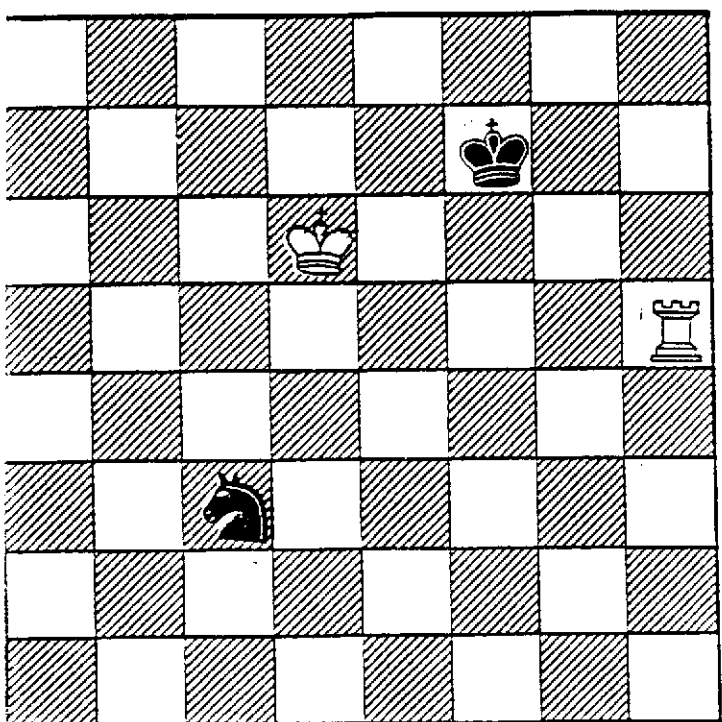(B to Move)

Diag. T

(B TC MOVE)
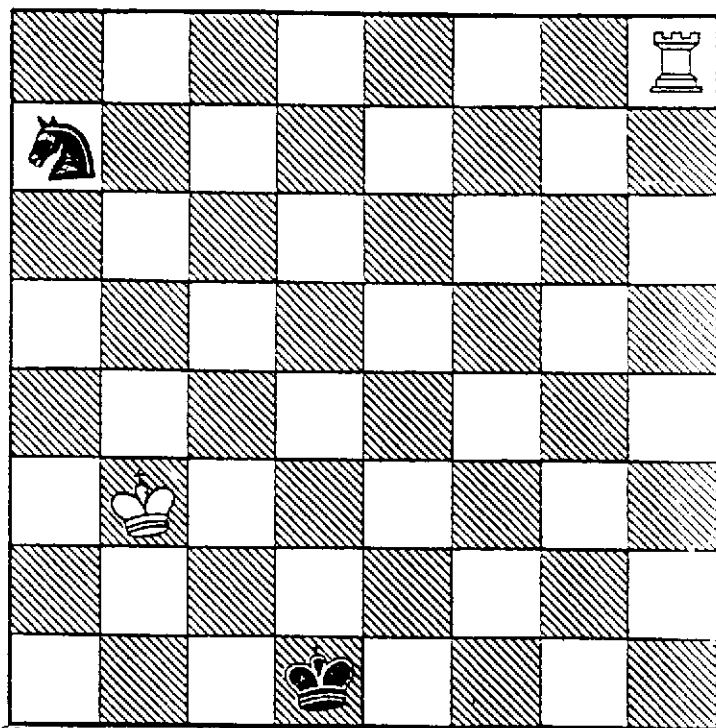
(B To Move)

(B to move)

(B to move)

Diag. D8.4

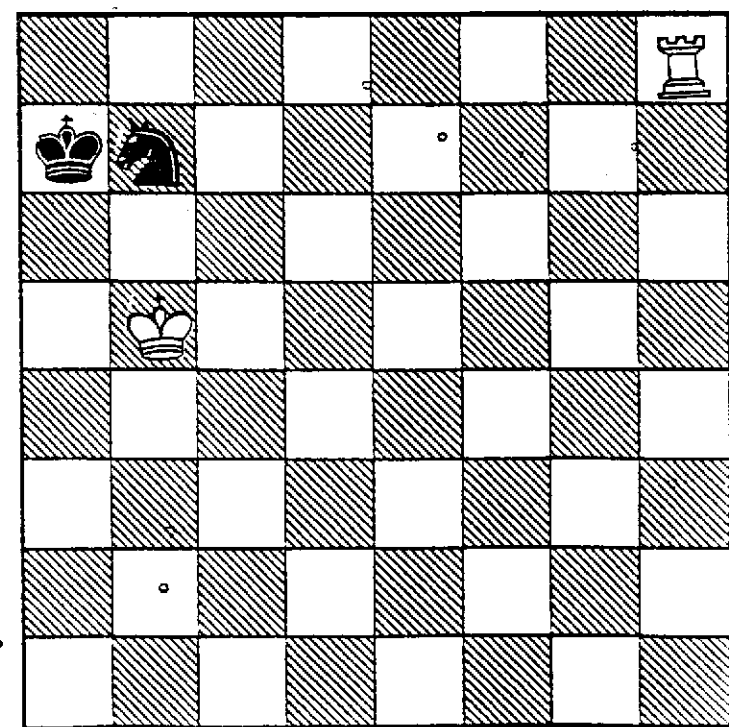Diag. D18

(W to move)

(B to move)

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